ASSIGNMENT SET - I

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

B.Sc Hon.(CBCS)

Mathematics: Semester-VI

Paper Code: DSE-3T

[Mechanics]

Answer all the questions

1. What are stable and unstable equilibrium? State energy test of stability.
2. State the principle of virtual work for a system of forces acting on a rigid body.
3. State Kepler's laws of planetary motion
4. If the angular velocity about the origin be a constant w, then find the cross –radial Component of rate of change of acceleration of the particle.
5. If X,Y be the algebraic sums of the resolved parts of all the forces acting in a plane along the rectangular axes θ_x *and* θ_y; and G be the algebraic sum of the moments of the forces about O; find the locus of the points at which the algebraic sum of the moments of the forces is constant and equal to G^1.
6. Define 'apse' of a central orbit. Show that, at an apse, a particle is moving at right angles to the radius vector of the point.
7. An artificial satellite revolves about the earth at a height H above the surface. Find the orbital speed, so that a man in the satellite will be in a state of weightlessness.
8. State D' Alembert's principle. Write down the general equations of motion of a rigid body.
9. Find the co- ordinates of C.G. of a lamina in the shape or a quadrant of the curve $(\frac{x}{a})^{\frac{2}{3}} + (\frac{y}{b})^{\frac{2}{3}}$ = 1 , density at(x, y) is ρ = *kxy* , where k is constant.
10. A square lamina rests with its plane perpendicular to a smooth wall, one corner being attached to a point in the wall by a fire string of length equal to the side of the square. Find the positive of equilibrium and show that it is stable.

11. At the vertex C of a triangle ABC which is right angled at C, show that the principal axes are perpendicular to the plane and two others inclined to the sides at an angle $\frac{1}{2}\tan^{-1}\frac{ab}{a^2-b^2}$.
12. An ellipse of axes a, b and a circle of radius b are cut from the same sheet of a uniform metal and are suspended and fixed together with their centres coincident. The figure is free to move in its own vertical plane about one end of its major axis . Show that the length of the equivalent simple pendulum is $\frac{5a^2-ab+2b^2}{4a}$.
13. A particle is projected at right angles to the line joining it to a centre of force, attracting according to the law of inverse square of the distance, with a velocity $\frac{\sqrt{3}}{2}v$, where v denotes the velocity from infinity. Find the eccentricity of the orbit described and show that the periodic time is 2π; T being the time taken to describe the major axis of the orbit with velocity V.
14. Find the accelerations of a particle, moving in 3-dimensional space, in terms of polar co – ordinates.
15. The middle points of opposite sides of a quadrilateral formed by four freely jointed weightless bars are connected by two light rods of length 'a' and 'b' in a state of tension. If $\mathrm{T_1}$ and $\mathrm{T_2}$ be the tensions of those rods, prove that $\frac{\mathrm{T_1}}{a}+\frac{\mathrm{T_2}}{b}$ = 0.
16.) A surface is formed by revolution of rectangular hyperbola about a vertical asymptote; show that a particle will rest on it everywhere beyond its intersection with a certain circular cylinder.
17.) If X, Y, Z, L, M, N are six components of a system of forces, deduce the invariants of the system.
18. Equal forces act along the axes and along the straight line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$.
19. Find the equation of the central axis of the system.
20. A particle moves with a central acceleration $\{\mu \div (distance)^2\}$. It is projected with a velocity V at a distance R. shoe that its path is a rectangular hyperbola, if the angle of projection is

$$\left\{\frac{\mu}{VR\sqrt{V^2-\frac{2\mu}{R}}}\right\}.$$

21. One of an elastic string of upstretched length 'a', is tied to a point on a smooth table and a particle is attached 6to the other end and can move freely on the table. If the path be nearly a circle of radius b, then show that apsidal angle is approximately $\pi\sqrt{\frac{(b-a)}{4b-3a}}$.
22. A thin rod of length 2a revolves with uniform angular velocity ω about a vertical axis through a small joint at one extremity of the rod, so that it describes a cone of semi- vertical angle α . Show that t$\omega^2 = \frac{3g}{4a\cos\alpha}$.
23. An elliptic lamina is such that when it swings about one latus rectum as a horizontal axis, the other latus rectum passes through the centre of oscillation. Prove that the eccentricity of the ellipse is $\frac{1}{2}$.

24. A particle of unit mass is projected with velocity u at an inclination α about the horizon in a medium whose resistance is k times the velocity. Show that the direction of path described will again make an angle α with the horizon after a time
$$\frac{1}{k}\log\left\{1+\frac{2ku}{g}\sin\alpha\right\}$$
25. Find the apsidal angle in a nearly circular orbit under the central force $ar^m + br^n$; a, b are constants.
26. Find the kinetic energy of a body moving in two dimensions.
27. A lamina in the form of an ellipse is rotating in its own plane about one of its foci with angular velocity ω. This focus is set free and the other, at the same instant is fixed. Show that the ellipse now rotate about it with angular $\omega\frac{2-5e^2}{2+3e^2}$.
28. Having given the moments and products of inertia of a rigid about three perpendicular concurrent axes. Find the moment of inertia of the body about an axis, with known direction cosines through that
29. A beam of length l rests with its ends on two smooth planes which intersect in a horizontal line. If the inclinations of the planes to the horizontal are α and β , and the center of gravity of the beam divides it in the ratio a: b. Find the position of the beam and show that the equilibrium is unstable.
30. If a hemisphere rests in equilibrium with its curved surface in contact with a rough plane inclined to a horizontal at an angle θ then show that the inclination of the plane of the hemisphere to the horizontal is $\sin^{-1}(\frac{8}{3}\sin\theta)$, provided $\vartheta<\sin^{-1}\frac{3}{8}$.
31. Prove that every given system of forces acting on a rigid body can be reduced to a wrench.
32. Six forces each equal to P, act along the edges of a cube, taken in order which do not meet a given diagonal. Show that their resultant is a couple of moment$2\sqrt{3}$ pa , where a is the edge of the cube.

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